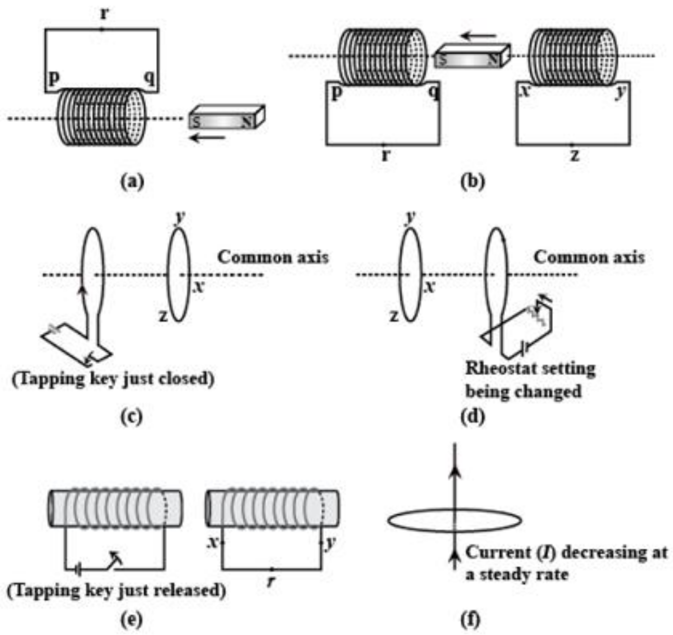
EXERCISE QUESTIONS

CHAPTER - 6 Electromagnetic Induction

6.1 Predict the direction of induced current in the situations described by the following Figs. 6.18(a) to (f).



Ans - (A) According to Lenz's law, the coil's face facing the magnet's south pole opposes that pole. Therefore, this face should act as the south pole. As a result, the current moves along qrpq.

(b) The current flows along yzxy and along prqp for a similar reason to the one stated previously.

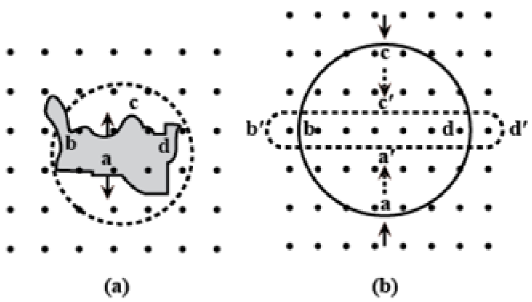
(c) When the coil is activated, the cell's growing current causes an adjacent coil along yzxy to experience an inverted current.

(d) The current flows along zyvz for a similar reason to that stated previously.

(e) According to Lenz's law, the current runs along xryx.

(f) Because field lines are in the plane of the loop, no induced current exists.

6.2 Use Lenz's law to determine the direction of induced current in the situations described by Fig. 6.19:
(a) A wire of irregular shape turning into a circular shape;
(b) A circular loop being deformed into a narrow straight wire.



Ans - (a) As a result of a shape change, area grows, which in turn causes a corresponding rise in magnetic flux. Lenz's law is used to set up an induced current in the circular wire that rotates counterclockwise to create opposing flux. Because of this, the magnetic field is pointed upward.

(b) When a circular loop is transformed into a straight wire, its area reduces, which causes the magnetic flux that is connected to it to do the same. The magnetic field is hence upward and an induced current is set up in an anticlockwise manner.

6.3 A long solenoid with 15 turns per cm has a small loop of area 2.0 cm2 placed inside the solenoid normal to its axis. If the current carried by the solenoid changes steadily from 2.0 A to 4.0 A in 0.1 s, what is the induced emf in the loop while the current is changing?

Ans - The solenoid has a small loop of area, $A = 2.0\ cm^2 = 2 \times 10^{-4}\ m^2$

Current carried by the solenoid changes from 2 A to 4 A.

Change in current in the solenoid, di = 4 − 2 = 2 A

$$|\varepsilon| = \frac{d\phi_B}{dt} = \frac{d}{dt}(BA) = A\frac{dB}{dt}$$

since $B = \mu_0 n I$

$$\therefore \quad |\varepsilon| = A\frac{d}{dt}(\mu_0 n I) = \mu_0 n A \frac{dI}{dt}$$

Here, $n = 15/\text{cm} = 1500/\text{m},\ A = 2{\cdot}0\ \text{cm}^2 = 2\times10^{-4}\ \text{m}^2$

$$\frac{dI}{dt} = \frac{2}{0{\cdot}1} = 20\ \text{A s}^{-1}$$

$$\therefore \quad |\varepsilon| = 4\pi \times 10^{-7} \times 1500 \times 2 \times 10^{-4} \times 20 = 7{\cdot}5\times10^{-6}\text{V}$$

6.4 A rectangular wire loop of sides 8 cm and 2 cm with a small cut is moving out of a region of uniform magnetic field of magnitude 0.3 T directed normal to the loop. What is the emf developed across the cut if the velocity of the loop is 1 cm s–1 in a direction normal to the

(a) longer side,

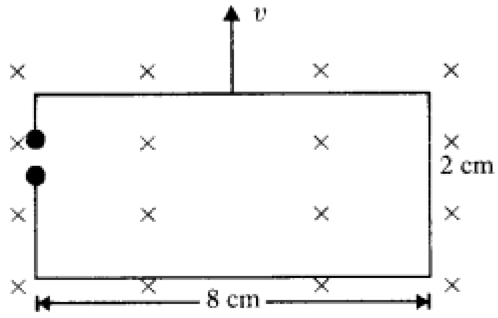
(b) shorter side of the loop?

For how long does the induced voltage last in each case?

Ans - Length of the rectangular wire, l = 8 cm = 0.08 m

Width of the rectangular wire, b = 2 cm = 0.02 m

$$\varepsilon = 0{\cdot}3 \times 8 \times 10^{-2} \times 10^{-2}$$
$$= 0{\cdot}24 \text{ mV}$$

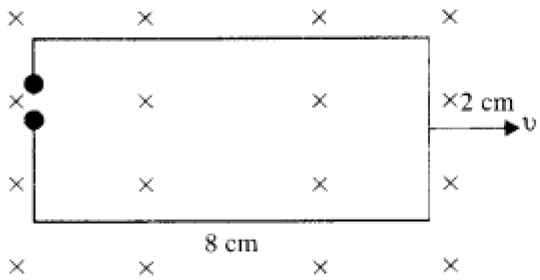


Time for which *e.m.f.* lasts

= time during which shorter side of rectangle moves out of the filed

$$= \frac{l}{v} = \frac{2\,\text{cm}}{1\,\text{cm s}^{-1}} = 2\text{s}$$

(*b*) $$|\varepsilon| = B\,l\,v = 0{\cdot}3 \times 2 \times 10^{-2} \times 10^{-2}$$
$$= 0{\cdot}06\ m\text{V}$$



Time for which *emf* lasts

= time during which longer side of rectangle moves out of the filed

$$= \frac{l}{v} = \frac{8}{1\,\text{cms}^{-1}} = 8 \text{ s.}$$

6.5 A 1.0 m long metallic rod is rotated with an angular frequency of 400 rad s–1 about an axis normal to the rod passing through its one end. The other end of the rod is in contact with a circular metallic ring. A constant and uniform magnetic field of 0.5 T parallel to the axis exists everywhere. Calculate the emf developed between the centre and the ring.

Ans -Length of the rod, l = 1 m

Angular frequency, ω = 400 rad/s

Magnetic field strength, B = 0.5 T

$$\text{Using} \quad \varepsilon = \frac{1}{2} B l^2 \omega$$

$$= \frac{0 \cdot 5 \times 1 \times 1 \times 400}{2} = 100 \text{ V}$$

6.6 A circular coil of radius 8.0 cm and 20 turns is rotated about its vertical diameter with an angular speed of 50 rad s–1 in a uniform horizontal magnetic field of magnitude 3.0 × 10–2 T. Obtain the maximum and average emf induced in the coil. If the coil forms a closed loop of resistance 10 Ω, calculate the maximum value of current in the coil. Calculate the average power loss due to Joule heating. Where does this power come from?

Ans - Max induced emf = 0.603 V

Average induced emf = 0 V

Max current in the coil = 0.0603 A

Average power loss = 0.018 W

(i) The induced *emf* in a rotating coil is given by

$$e = NBA\omega \sin \omega t$$

$$\therefore \quad e_{max} = NBA\omega = NB \times \pi r^2 \omega$$

$$= 20 \times 3 \times 10^{-2} \times 3{\cdot}14 \times 64 \times 10^{-4} \times 500$$

$$= 0{\cdot}603 \text{ V}$$

(ii) Since average value of sin ωt over a complete cycle is zero, therefore, average *emf* = zero.

(iii) $$I_{max} = \frac{e_{max}}{R} = \frac{0 \cdot 063}{10} = 0{\cdot}0603 \text{ A}$$

(iv) $$\text{Power loss} = \frac{1}{2} e_{max} I_{max}$$

$$= \frac{1}{2} \times 0{\cdot}603 \times 0{\cdot}0603$$

$$= \frac{0 \cdot 036}{2} = 0 \cdot 018 \text{ W}$$

6.7 A horizontal straight wire 10 m long extending from east to west is falling with a speed of 5.0 m s–1, at right angles to the horizontal component of the earth's magnetic field, 0.30 × 10–4 Wb m–2 .
(a) What is the instantaneous value of the emf induced in the wire? (b) What is the direction of the emf?
(c) Which end of the wire is at the higher electrical potential?
Ans - (a) Instantaneous emf ε = Bυl

$$\varepsilon = 0.3 \times 10^{-4} \times 5 \times 10$$
$$= 15 \times 10^{-4} \text{ volt} = 1.5 \text{ mV}$$

(b) According to Fleming's Right-Hand Rule, the induced electromagnetic field is directed from west to east.
(c) The Western end of the rod will have a higher electrical potential since it will serve as a source.

6.8 Current in a circuit falls from 5.0 A to 0.0 A in 0.1 s. If an average emf of 200 V induced, give an estimate of the self-inductance of the circuit.
Ans - Initial current, I_1 = 5.0 A

Final current, I_2 = 0.0 A

$$\text{Using } |\varepsilon| = L\frac{di}{dt}, \text{ we get}$$

$$20x = L\left(\frac{5-0}{0\cdot 1}\right)$$

6.9 A pair of adjacent coils has a mutual inductance of 1.5 H. If the current in one coil changes from 0 to 20 A in 0.5 s, what is the change of flux linkage with the other coil?
Ans - Mutual inductance, M = 1.5 H

Current in the coil, I = 20 A

dΦB = M.dl = 1.5 x (20 – 0) = 30 Wb

6.10 A jet plane is travelling towards west at a speed of 1800 km/h. What is the voltage difference developed between the

ends of the wing having a span of 25 m, if the Earth's magnetic field at the location has a magnitude of 5 × 10–4 T and the dip angle is 30°.

Ans - Wing span of the jet, l = 25 m

Magnetic field strength by earth,

$$\text{Using} \quad e = B_\upsilon l \upsilon, \text{ we get } e = (B \sin 30°) l \upsilon$$
$$e = 500 \sin 30° \, (5 \times 10^{-4}) \, 25$$
$$= \frac{500}{2} \times 5 \times 10^{-4} \times 25 = 3{\cdot}1 \text{ V}$$

6.11 Suppose the loop in Exercise 6.4 is stationary but the current feeding the electromagnet that produces the magnetic field is gradually reduced so that the field decreases from its initial value of 0.3 T at the rate of 0.02 T s –1. If the cut is joined and the loop has a resistance of 1.6 Ω, how much power is dissipated by the loop as heat? What is the source of this power?

Ans - Sides of the rectangular loop are 8 cm and 2 cm.

area of the rectangular wire loop,

$$\text{Induced e.m.f., } e = \frac{d\phi}{dt} = A\frac{dB}{dt}$$
$$= (0{\cdot}02)\,(8 \times 2 \times 10^{-4})$$
$$= 3{\cdot}2 \times 10^{-5} \text{ V}$$

$$\text{Induced current, } I = \frac{e}{R} = \frac{3 \cdot 2}{1 \cdot 6} \times 10^{-5}$$
$$= 2 \times 10^{-5} \text{ A}$$
$$\text{Power loss} = I^2 R$$
$$= 4 \times 10^{-10} \times 1{\cdot}6$$
$$= 6{\cdot}4 \times 10^{-10} \text{ W}$$

6.12 A square loop of side 12 cm with its sides parallel to X and Y axes is moved with a velocity of 8 cm s –1 in the positive x-direction in an environment containing a magnetic

field in the positive z-direction. The field is neither uniform in space nor constant in time. It has a gradient of 10 –3 T cm–1 along the negative x-direction (that is it increases by 10 – 3 T cm–1 as one moves in the negative x-direction), and it is decreasing in time at the rate of 10 –3 T s–1. Determine the direction and magnitude of the induced current in the loop if its resistance is 4.50 mΩ.

Ans - Side of the square loop, s = 12 cm = 0.12 m

Area of the square loop, A = 0.12 × 0.12 = 0.0144 m^2

Velocity of the loop, v = 8 cm/s = 0.08 m/s

$$\frac{dB}{dt} = 10^{-3} \text{ T s}^{-1}$$

Total induced e.m.f./'e

$$= emf \text{ due to } \frac{dB}{dx} + e.m.f. \text{ due to } \frac{dB}{dt}$$

$$= \frac{d}{dt}(BA) + \frac{d}{dt}(BA)$$

$$= A\frac{dB}{dt} + A\frac{dB}{dt} = A\frac{dB}{dx}\cdot\frac{dx}{dt} + A\frac{dB}{dt}.$$

$$= Av\frac{dB}{dx} + A\frac{dB}{dt} \qquad (\because v = \frac{dx}{dt})$$

$$= 144 \times 10^{-4} \times 8 \times 10^{-2} \times 10^{-1} + 144 \times 10^{-4} \times 10^{-3}$$

$$= 115{\cdot}2 \times 10^{-6} + 14{\cdot}4 \times 10^{-6}$$

$$= 129{\cdot}6 \times 10^{-6} \text{ V}$$

$$\text{Induced current, } I = \frac{e}{R} = \frac{129{\cdot}6 \times 10^{-6}}{4{\cdot}50 \times 10^{-3}} = 2{\cdot}88 \times 10^{-2} \text{ A}$$

6.13 It is desired to measure the magnitude of field between the poles of a powerful loud speaker magnet. A small flat search coil of area 2 cm2 with 25 closely wound turns, is positioned normal to the field direction, and then quickly snatched out of the field region. Equivalently, one can give it a quick 90° turn to bring its plane parallel to the field direction).

The total charge flown in the coil (measured by a ballistic galvanometer connected to coil) is 7.5 mC. The combined resistance of the coil and the galvanometer is 0.50 Ω. Estimate the field strength of magnet.

Ans - A small flat search coil's surface area is equal to 2 cm2 or 2 104 m2.

N is the coil's turn count, which is 25.

Using $$e = -N\frac{d\phi}{dt} = -N\frac{dB}{dt}A$$

But $$e = IR \quad \therefore \quad IR = -N\frac{dB}{dt}A$$

$$i.e.\ R\frac{dq}{dt} = -\frac{NA\,dB}{dt}$$

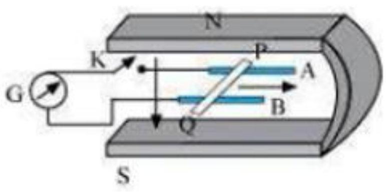
$$i.e.\ R\frac{dq}{dt} = -\frac{NA\,dB}{dt} \quad \text{or} \quad Rq = NAB$$

or $$B = \frac{Rq}{NA}$$

$$= \frac{0\cdot 5 \times 7\cdot 5 \times 10^{-3}}{25 \times 2 \times 10^{-4}} = 0\cdot 75\ T$$

6.14 Figure 6.20 shows a metal rod PQ resting on the smooth rails AB and positioned between the poles of a permanent magnet. The rails, the rod, and the magnetic field are in three mutual perpendicular directions. A galvanometer G connects the rails through a switch K. Length of the rod = 15 cm, B = 0.50 T, resistance of the closed loop containing the rod = 9.0 mΩ. Assume the field to be uniform.

(a) Suppose K is open and the rod is moved with a speed of 12 cm s–1 in the direction shown. Give the polarity and magnitude of the induced emf.



(b) Is there an excess charge built up at the ends of the rods when K is open? What if K is closed?
(c) With K open and the rod moving uniformly, there is no net force on the electrons in the rod PQ even though they do experience magnetic force due to the motion of the rod. Explain.
(d) What is the retarding force on the rod when K is closed?
(e) How much power is required (by an external agent) to keep the rod moving at the same speed (=12 cm s–1) when K is closed? How much power is required when K is open?
(f) How much power is dissipated as heat in the closed circuit? What is the source of this power?
(g) What is the induced emf in the moving rod if the magnetic field is parallel to the rails instead of being perpendicular?
Ans -

(a) Switch K is opened, and the rod moves in three mutually perpendicular directions at a speed of 12 cm-1.
Motional emf/induced emf =

Bl = 0.5 12 10-2 15 10-2
= 9 mV

(b) The rod's top end becomes positively charged and its lower end becomes negatively charged when the K is open.

Charge flows in a closed circuit when the K is closed, but the extra charge is retained by the flow of charge in the moving rod while it is being pulled by a magnetic field.

(c) The electronic force operating on the electron due to the electronic field created across the two ends as a result of the accumulation of positive and negative charges at the ends P and Q, respectively, cancels the magnetic Lorentz force acting on it.

(d) The magnetic field of the current-carrying wire experiences a retarding effect when the key is closed, causing the current to run in a loop.

$$\text{Here,} \quad B = \mu_0 \frac{N}{l} I$$

$$= (4\pi \times 10^{-7}) \frac{500}{(30 \times 10^{-2})} 2{\cdot}5$$

$$= 52{\cdot}36 \times 10^{-4} \text{ T}$$

Initial flux linkage

$$\phi = BAN = 52{\cdot}36 \times 10^{-4} \times 25 \times 10^{-4} \times 500$$

$$= 65{\cdot}45 \times 10^{-4} \text{ Wb}$$

Final flux linkage

$$\phi_f = 0$$

$$d\phi = -\,65{\cdot}45 \times 10^{-4} \text{ Wb}$$

$$\text{Then,} \quad e = -\frac{d\phi}{dt} = \frac{65{\cdot}45 \times 10^{-4}}{10^{-3}} = 6{\cdot}54 \text{ V} \approx 6{\cdot}5 \text{ V.}$$

6.15 An air-cored solenoid with length 30 cm, area of cross-section 25 cm2 and number of turns 500, carries a current of 2.5 A. The current is suddenly switched off in a brief time of 10–3 s. How much is the average back emf induced across the ends of the open switch in the circuit? Ignore the variation in magnetic field near the ends of the solenoid.

Ans - Length of the solenoid, l = 30 cm = 0.3 m

Area of cross-section, A = 25 cm^2 = 25 × 10^{-4} m

$$\text{Here,} \quad B = \mu_0 \frac{N}{l} I$$

$$= (4\pi \times 10^{-7}) \frac{500}{(30 \times 10^{-2})} 2{\cdot}5$$

$$= 52{\cdot}36 \times 10^{-4} \text{ T}$$

Initial flux linkage

$$\phi = BAN = 52{\cdot}36 \times 10^{-4} \times 25 \times 10^{-4} \times 500$$

$$= 65{\cdot}45 \times 10^{-4} \text{ Wb}$$

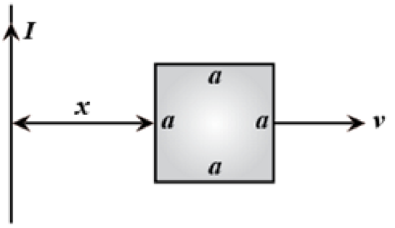
Final flux linkage

$$\phi_f = 0$$

$$d\phi = -\,65{\cdot}45 \times 10^{-4} \text{ Wb}$$

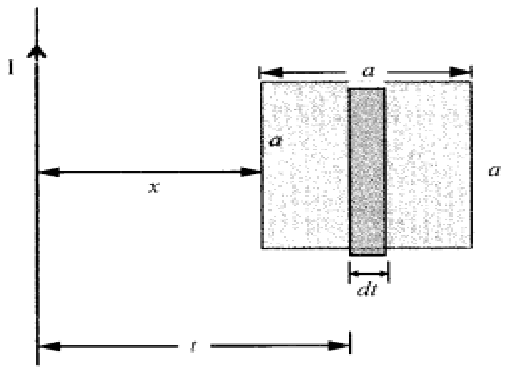
$$\text{Then,} \quad e = -\frac{d\phi}{dt} = \frac{65{\cdot}45 \times 10^{-4}}{10^{-3}} = 6{\cdot}54 \text{ V} \approx 6{\cdot}5 \text{ V.}$$

6.16 (a) Obtain an expression for the mutual inductance between a long straight wire and a square loop of side a as shown in Fig. 6.21.
(b) Now assume that the straight wire carries a current of 50 A and the loop is moved to the right with a constant velocity, v = 10 m/s. Calculate the induced emf in the loop at the instant when x = 0.2 m. Take a = 0.1 m and assume that the loop has a large resistance.



Ans - (A) Take into account a small section of the coil with thickness dt that is located t distances away from the current-carrying wire. thus, the intensity of the magnetic field felt by this area

$$B = \frac{\mu_0 I}{2\pi t}$$



Area of the strip, $dA = a\,.dt$

$\therefore$ Magnetic flux linked with the strip,

$$d\phi = BdA = \frac{\mu_0 Ia}{2\pi t}dt$$

$\therefore$ Total magnetic flux linked with the coil (within limits from $t = x$ to $t = a + x$) is given by

$$\phi = \int_x^{a+x} \frac{\mu_0 Ia}{2\pi t}dt = \frac{\mu_0 Ia}{2\pi}\int_x^{a+x} \frac{1}{t}dt$$

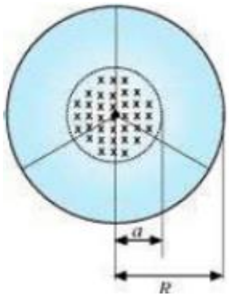
$$= \frac{\mu_0 Ia}{2\pi}[\log_e t]_x^{a+x}$$

(b) Emf induced in the loop, e = B'av = $\left(\frac{\mu_0 I}{2\pi x}\right)av$

$$e = \frac{4\pi \times 10^{-7} \times 50 \times 0.1 \times 10}{2\pi \times 0.2}$$

$$e = 5 \times 10^{-5}\ V$$

6.17 A line charge λ per unit length is lodged uniformly onto the rim of a wheel of mass M and radius R. The wheel has light non-conducting spokes and is free to rotate without friction about its axis (Fig. 6.22). A uniform magnetic field extends over a circular region within the rim. It is given by, $B = -B_0 k$ ($r \le a$; $a < R$) = 0 (otherwise) What is the angular velocity of the wheel after the field is suddenly switched off?



Ans -

Where,

r = Distance of the point within the wheel

Mass of the wheel = M

Radius of the wheel = R

$$\frac{d\vec{B}}{dt} = \frac{B_0}{t}\hat{k}$$

The work done in moving the charge once around the loop due to electric field E produced by the magnetic field is given by

$$W = F \times g = qE\,(2\pi r)$$

$$\therefore \qquad \varepsilon = \frac{W}{q} = E\,.\,2\,\pi\,R \qquad \ldots(i)$$

But, from Faraday's law of electromagnetic induction,

$$\varepsilon = -\pi a^2 \frac{dB}{dt} \qquad \ldots(ii)$$

Equating (*i*) and (*ii*),

$$E.2\pi R = -\pi a^2 \frac{dB}{dt}.$$

Multiplying both sides by $q\,dt$ and rearranging, we get

$$qEdt = -\frac{\pi a^2 q}{2\pi R}dB \qquad \ldots(iii)$$

But $qEdt = F\,dt$ = Impulse = Change in momentum of the wheel.

$$= (M\upsilon - 0) = M\upsilon = M\omega R$$

$\therefore$ from eqn. (*iii*),

$$M\omega R = -\pi a^2\left(\frac{q}{2\pi R}\right)dB.$$

But $\dfrac{q}{2\pi R} = \lambda$ and $dB = B_0$ (in magnitude)